

Signs of quadratic function

Part 3



Quadratic functions

In the previous videos, We've learned how to determine the sign of a quadratic function.

In this part, we will learn about the signs of the roots of a quadratic function.



Signs of roots of a quadratic functions

Consider the quadratic function: $f(x) = ax^2 + bx + c ; a \neq 0$

It is possible to determine the signs of the roots of the equation $f(x) = 0$ without solving it.



Signs of roots of a quadratic functions

Consider the quadratic function: $f(x) = ax^2 + bx + c ; a \neq 0$

It is possible to determine the signs of the roots of the equation $f(x) = 0$ without solving it.

We need to check the existence of the roots in \mathbb{R}

$$\Delta > 0$$

❖ Two distinct roots.

$$\Delta = 0$$

❖ One double root

$$\Delta < 0$$

No real roots

No signs



Signs of roots of a quadratic functions

Case 1: $\Delta > 0$

To study the signs of the roots, we need:

Sum S

and

Product P

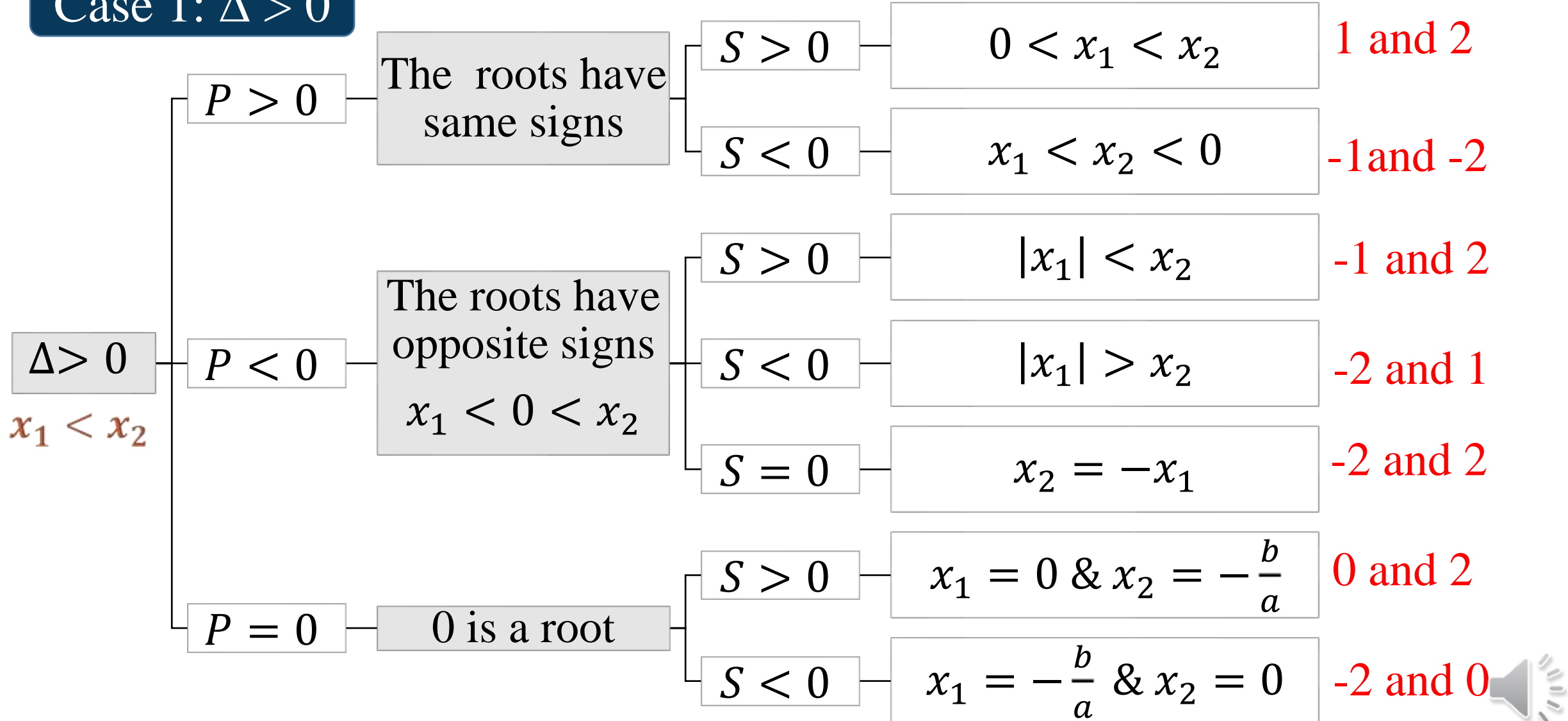
$$S = x_1 + x_2 = -\frac{b}{a}$$

$$P = x_1 \cdot x_2 = \frac{c}{a}$$



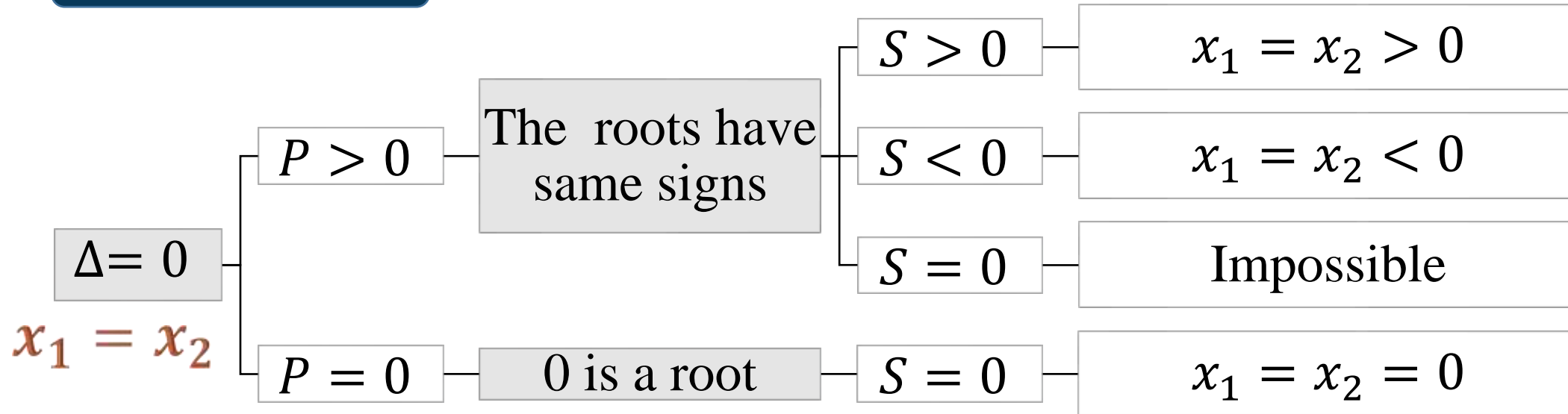
Signs of roots of a quadratic functions

Case 1: $\Delta > 0$



Signs of roots of a quadratic functions

Case 2: $\Delta = 0$



In this case
 $f(x) = ax^2$



Signs of roots of a quadratic functions

Example 1

Study the signs of the roots.

$$f(x) = 2x^2 - x + 1$$

$$\Delta = b^2 - 4ac = (-1)^2 - 4(2)(1) = 1 - 8 = -7 < 0$$

So the roots doesn't exist in IR



Signs of roots of a quadratic functions

Example 2

Study the signs of the roots.

$$f(x) = 4x^2 - 4x + 1$$

$$\Delta = b^2 - 4ac = (-4)^2 - 4(4)(1) = 16 - 16 = 0 \text{ so } x_1 = x_2$$

$$P = \frac{c}{a} = \frac{1}{4} > 0 \text{ so the two roots are positive or negative}$$

$$S = -\frac{b}{a} = \frac{4}{4} = 1 > 0 \text{ so the two roots are positive}$$



Signs of roots of a quadratic functions

Example 3

Study the signs of the roots.

$$f(x) = 5x^2 - 3x - 2$$

$$\Delta = b^2 - 4ac = (-3)^2 - 4(5)(-2) = 9 + 40 = 47 > 0$$

So the two roots are distinct: $x_1 < x_2$

$$P = \frac{c}{a} = -\frac{2}{5} < 0 \text{ so the two roots are of opposite signs: } x_1 < 0 < x_2$$

$$S = -\frac{b}{a} = \frac{3}{5} > 0 \text{ so } |x_1| < x_2$$



Signs of roots of a quadratic functions

Example 4

Study the signs of the roots.

$$f(x) = 3x^2 - 5x + 1$$

$$\Delta = b^2 - 4ac = (-5)^2 - 4(3)(1) = 25 - 12 = 13 > 0$$

So the two roots are distinct: $x_1 < x_2$

$$P = \frac{c}{a} = \frac{1}{3} > 0 \text{ so the two roots have same signs.}$$

$$S = -\frac{b}{a} = \frac{5}{3} > 0 \text{ so } 0 < x_1 < x_2$$



Signs of roots of a quadratic functions

Example 5

Study the signs of the roots in each case.

$$f(x) = x^2 + 2x$$

$$\Delta = b^2 - 4ac = (2)^2 - 4(1)(0) = 4 - 0 = 4 > 0$$

So the two roots are distinct: $x_1 < x_2$

$$P = \frac{c}{a} = \frac{0}{1} = 0 \text{ so } 0 \text{ is one of the roots}$$

$$S = -\frac{b}{a} = \frac{-2}{1} = -2 < 0 \text{ so } x_1 = 0 \text{ \& } x_2 = -\frac{b}{a} = -2$$



Time for practice

Study the signs of the roots in each case.

$$1) f(x) = 2x^2 - 5x - 1$$

$$x_1 < 0 < x_2 ; |x_1| > x_2$$

$$2) f(x) = -3x^2 + 4x - 8$$

No real roots

$$3) f(x) = 7x^2 - 12x + 5$$

$$x_1 < x_2 < 0$$

$$4) f(x) = -x^2 + 2x + 1$$

$$0 < x_1 < x_2$$

$$5) f(x) = x^2 - x + \frac{1}{4}$$

$$x_1 = x_2 < 0$$



