Signs of quadratic function

Part 3

Quadratic functions

In the previous videos, We've learned how to determine the sign of a quadratic function.
In this part, we will learn about the signs of the roots of a quadratic function.

Signs of roots of a quadratic functions

Consider the quadratic function: $f(x) = ax^2 + bx + c$; $a \neq 0$

It is possible to determine the signs of the roots of the equation $f(x) = 0$ without solving it.

Signs of roots of a quadratic functions

Consider the quadratic function: $f(x) = ax^2 + bx + c$; $a \neq 0$

It is possible to determine the signs of the roots of the equation $f(x) = 0$ without solving it.

We need to check the existence of the roots in IR

$\Delta > 0$	$\Delta = 0$	$\Delta < 0$
❖ Two distinct roots.	❖ One double root	No real roots

No signs

Signs of roots of a quadratic functions

Case 1: $\Delta > 0$

To study the signs of the roots, we need:

Sum S and **Product P**

$$S = x_1 + x_2 = -\frac{b}{a}$$

$$P = x_1 . x_2 = \frac{c}{a}$$

Signs of roots of a quadratic functions

Case 1: $\Delta > 0$

$\Delta > 0$

$x_1 < x_2$

$P > 0$	The roots have same signs	$S > 0$	$0 < x_1 < x_2$	1 and 2
		$S < 0$	$x_1 < x_2 < 0$	-1and -2
$P < 0$	The roots have opposite signs $x_1 < 0 < x_2$	$S > 0$	$\lvert x_1 \rvert < x_2$	-1 and 2
		$S < 0$	$\lvert x_1 \rvert > x_2$	-2 and 1
		$S = 0$	$x_2 = -x_1$	-2 and 2
$P = 0$	0 is a root	$S > 0$	$x_1 = 0 \,\&\, x_2 = -\frac{b}{a}$	0 and 2
		$S < 0$	$x_1 = -\frac{b}{a} \,\&\, x_2 = 0$	-2 and 0

Signs of roots of a quadratic functions

Case 2: $\Delta = 0$

$\Delta = 0$

$x_1 = x_2$

- $P > 0$ — The roots have same signs
  - $S > 0$ — $x_1 = x_2 > 0$
  - $S < 0$ — $x_1 = x_2 < 0$
  - $S = 0$ — Impossible
- $P = 0$ — 0 is a root — $S = 0$ — $x_1 = x_2 = 0$

In this case

$f(x) = ax^2$

Signs of roots of a quadratic functions

Example 1

Study the signs of the roots.

$f(x) = 2x^2 - x + 1$

$\Delta = b^2 - 4ac = (-1)^2 - 4(2)(1) = 1 - 8 = -7 < 0$

So the roots doesn't exist in IR

Signs of roots of a quadratic functions

Example 2

Study the signs of the roots.

$f(x) = 4x^2 - 4x + 1$

$\Delta = b^2 - 4ac = (-4)^2 - 4(4)(1) = 16 - 16 = 0$ so $x_1 = x_2$

$P = \frac{c}{a} = \frac{1}{4} > 0$ so the two roots are positive or negative

$S = -\frac{b}{a} = \frac{4}{4} = 1 > 0$ so the two roots are positive

Signs of roots of a quadratic functions

Example 3

Study the signs of the roots.

$f(x) = 5x^2 - 3x - 2$

$\Delta = b^2 - 4ac = (-3)^2 - 4(5)(-2) = 9 + 40 = 47 > 0$

So the two roots are distinct: $x_1 < x_2$

$P = \frac{c}{a} = -\frac{2}{5} < 0$ so the two roots are of opposite signs: $x_1 < 0 < x_2$

$S = -\frac{b}{a} = \frac{3}{5} > 0$ so $|x_1| < x_2$

Signs of roots of a quadratic functions

Example 4

Study the signs of the roots.

$f(x) = 3x^2 - 5x + 1$

$\Delta = b^2 - 4ac = (-5)^2 - 4(3)(1) = 25 - 12 = 13 > 0$

So the two roots are distinct: $x_1 < x_2$

$P = \frac{c}{a} = \frac{1}{3} > 0$ so the two roots have same signs.

$S = -\frac{b}{a} = \frac{5}{3} > 0$ so $0 < x_1 < x_2$

Signs of roots of a quadratic functions

Example 5

Study the signs of the roots in each case.

$f(x) = x^2 + 2x$

$\Delta = b^2 - 4ac = (2)^2 - 4(1)(0) = 4 - 0 = 4 > 0$

So the two roots are distinct: $x_1 < x_2$

$P = \frac{c}{a} = \frac{0}{1} = 0$ so 0 is one of the roots

$S = -\frac{b}{a} = \frac{-2}{1} = -2 < 0$ so $x_1 = 0$ & $x_2 = -\frac{b}{a} = -2$

Time for practice

Study the signs of the roots in each case.

1) $f(x) = 2x^2 - 5x - 1$ — $x_1 < 0 < x_2$; $|x_1| > x_2$

2) $f(x) = -3x^2 + 4x - 8$ — No real roots

3) $f(x) = 7x^2 - 12x + 5$ — $x_1 < x_2 < 0$

4) $f(x) = -x^2 + 2x + 1$ — $0 < x_1 < x_2$

5) $f(x) = x^2 - x + \frac{1}{4}$ — $x_1 = x_2 < 0$